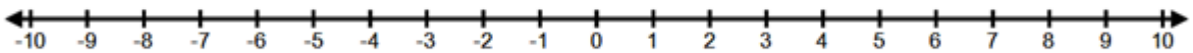


Lesson 6: Compound Inequalities

Example 1 – Solving an “Or” (Union of Sets) Compound Inequality

Solve the compound inequality and graph its solution on a number line.

$$x + 10 < 7 \quad \text{Or} \quad 2x - 5 > -3$$



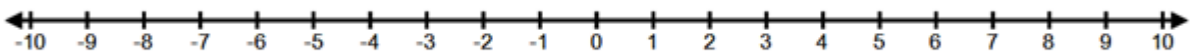
Example 2 – Solving an “And” (Intersection of Sets) Compound Inequality

Solve the inequality and graph the solution on a number line.

$$20 > -3x - 4 > 5$$

Method 1: Separate using “And”

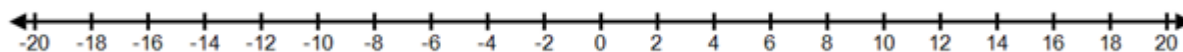
Method 2: Solve at the same time



Example 3

Solve and graph the solution on a number line:

$$4 < \frac{1}{2}(x - 8) + 1 < 6$$

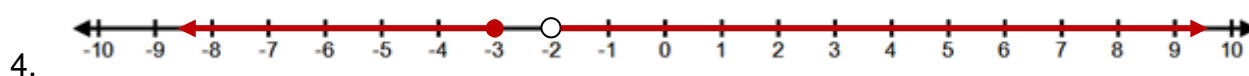
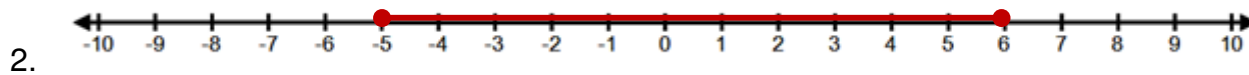
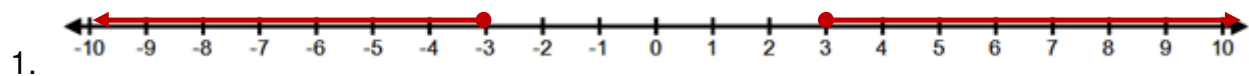


**Lesson 6: Compound Inequalities – Practice Problems**

**Part 1:** Write a compound inequality to describe the following situations.

1. Buy a present,  $p$ , that costs at least \$10 and at most \$25.
2. Don't cook the muffins less than 12 minutes or more than 15 minutes.
3. Training run must not be less than 10 miles or more than 15 miles.
4. Eat at least 1200 calories and no more than 2000 calories.

**Part 2:** Write a compound inequality for each graph.



**Part 3:** Solve each inequality and graph the solution on a number line.

1.  $-3x - 5 < -17$  or  $2x + 1 < -38$
2.  $26 \geq -4x + 2 \geq -2$
3.  $-7 < .4x - 5 < -3$
4.  $2(a-9) + 3 > -9$  or  $3(4x-5) < 9$
5.  $-8 > -3/4x - 2 > -11$

6.  $-5 > -4(a - 3) + 1 > -17$
7.  $\frac{2}{3}x + \frac{1}{6} > -5$  or  $\frac{1}{4}(x - 8) < -1$
8.  $22.8 < 1.5(x - 5) + .3 < 39.3$
9.  $-3(x + 8) - 7x > 46$  or  $4(x - 2) + 3x > 15$
10. Challenge:  $\frac{3}{4} < \frac{3}{4} + \frac{2}{3}(x - 5) < \frac{11}{4}$



For numbers 1-3, write a compound inequality for each and then graph the solution set on a number line. (2 points each)

1. all numbers greater than or equal to 8 or less than or equal to -2.
2. all numbers between -6 and 6.
3. a starting salary that's at least \$35000 and not more than \$55000.

For numbers 4 and 5, solve each compound inequality and graph the solution on a number line. (3 points each)

4.  $-\frac{2}{3}x + 7 < 3$  or  $5(x + 1) < 30$
5.  $-2 > -2(x - 5) > -4$

**Lesson 6: Compound Inequalities – Practice Problems**  
**Answer Key**

**Part 1:** Write a compound inequality to describe the following situations.

1. Buy a present,  $p$ , that costs at least \$10 and at most \$25.

$10 \leq p \leq 25$

10 is the least  $p$  can cost       $p$  is less than or equal to 25 or "at most" 25

2. Don't cook the muffins less than 12 minutes or more than 15 minutes.

$12 \leq m \leq 15$

12 is the least  $m$  can be       $m$  is less than or equal to 15 or "no more than"

3. Training run must not be less than 10 miles or more than 15 miles.

$10 \leq m \leq 15$

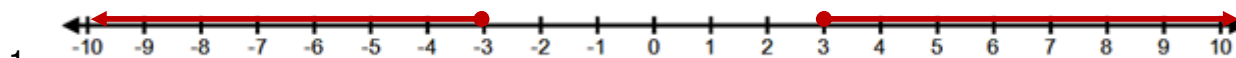
10 is the least  $m$  can be       $m$  is less than or equal to 15 or "no more than" 15

4. Eat at least 1200 calories and no more than 2000 calories.

$1200 \leq c \leq 2000$

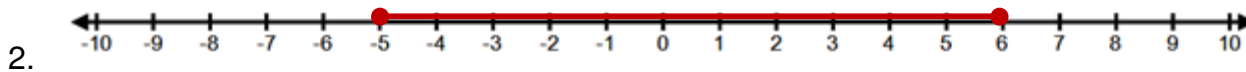
1200 is the least  $c$  can be       $c$  is less than or equal to 2000 or "no more than" 2000

**Part 2:** Write a compound inequality for each graph.



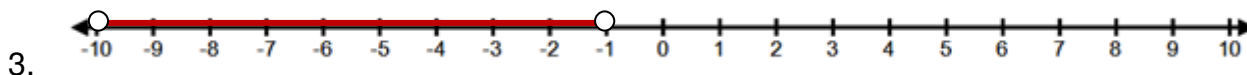
Since this graph contains two separate graphs, we know that his compound inequality is a union of sets joined by the word "Or".

$x \leq -3$  OR  $x \geq 3$



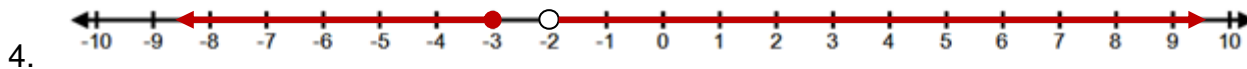
Since this graph contains one graph which is an intersection of sets, we know that we can use the word "And" or write it as one compound inequality.

$-5 \leq x \leq 6$  (  $x$  is between -5 and 6) Another way to write this  $\longrightarrow x \geq -5$  And  $x \leq 6$



Since this graph contains one graph which is an intersection of sets, we know that we can use the word "And" or write it as one compound inequality.

$-10 < x < -1$  ( x is between -10 and -1) Another way to write this  $\rightarrow x \geq -10$  And  $x \leq -1$



Since this graph contains two separate graphs, we know that his compound inequality is a union of sets joined by the word "Or".

$x \leq -3$  OR  $x > -2$

**Part 3:** Solve each inequality and graph the solution on a number line.

1.  $-3x - 5 < -17$  or  $2x + 1 < -38$

This is a union of sets, so we must solve each inequality separately.

$-3x - 5 < -17$

OR

$2x + 1 < -38$

$-3x - 5 + 5 < -17 + 5$  Add 5

$2x + 1 - 1 < -38 - 1$  Subtract 1

$-3x < -12$  Simplify

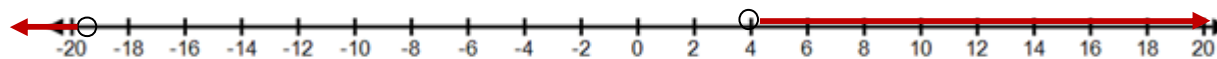
$2x < -39$  Simplify

$-3x / -3 < -12 / -3$  Divide by -3

$2x / 2 < -39 / 2$  Divide by 2

$x > 4$  Simplify

$x < -19.5$  Simplify



2.  $26 \geq -4x + 2 \geq -2$

$26 - 2 \geq -4x + 2 - 2 \geq -2 - 2$

Subtract 2 from all 3 sides.

$24 \geq -4x \geq -4$

Simplify

$24/-4 \geq -4x/-4 \geq -4/-4$

Divide by -4 on all 3 sides

$-6 \leq x \leq 1$

Simplify and reverse the inequality symbol



3.  $-7 < .4x - 5 < -3$

$-7 + 5 < .4x - 5 + 5 < -3 + 5$

Add 5 to all 3 sides

$-2 < .4x < 2$

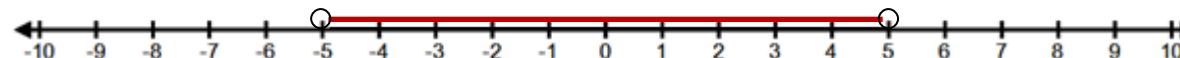
Simplify

$-2/.4 < .4x/.4 < 2/.4$

Divide by .4 on all 3 sides

$-5 < x < 5$

Simplify



4.  $2(a-9) + 3 > -9$  or  $3(4x-5) < 9$

This is a union of sets, so we must solve each inequality separately.

$2(a-9) + 3 > -9$

or

$3(4x-5) < 9$

$2a - 18 + 3 > -9$

Distribute

$12x - 15 < 9$

Distribute

$2a - 15 > -9$

Combine like terms

$12x - 15 + 15 < 9 + 15$

Add 15

$2x - 15 + 15 > -9 + 15$

Add 15 to both sides

$12x < 24$

Simplify

$2x > 6$

Simplify

$12x/12 < 24/12$

Divide by 12

$2x/2 > 6/2$

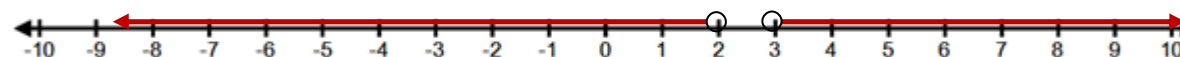
Divide by 2

$x < 2$

Simplify

$x > 3$

Simplify



5.  $-8 > -3/4x - 2 > -11$

$-8 + 2 > -3/4x - 2 + 2 > -11 + 2$

Add 2 to all 3 sides

$-6 > -3/4x > -9$

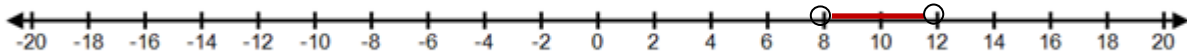
Simplify

$(-4/3)(-6) > (-4/3)(-3/4x) > (-4/3)(-9)$

Multiply by  $-4/3$  on all 3 sides

$8 < x < 12$

Simplify and reverse the sign



6.  $-5 > -4(a - 3) + 1 > -17$

$-5 > -4a + 12 + 1 > -17$

Distribute the  $-4$  throughout the parenthesis

$-5 > -4a + 13 > -17$

Combine like terms:  $12+1 = 13$

$-5 - 13 > -4a + 13 - 13 > -17 - 13$

Subtract 13 from all three sides

$-18 > -4a > -30$

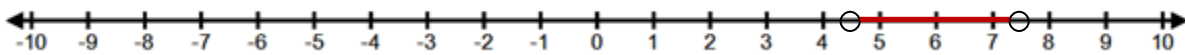
Simplify

$-18/-4 > -4a/-4 > -30/-4$

Divided by  $-4$  on all three sides

$4.5 < a < 7.5$

Simplify and reverse the sign



7.  $2/3x + 1/6 > -5$  or  $1/4(x-8) < -1$

This is a union of sets, so we must solve each inequality separately.

$2/3x + 1/6 > -5$

OR

$1/4(x-8) < -1$

$6[2/3x + 1/6] > -5(6)$

Multiply by 6

$4[1/4(x-8)] < -1(4)$

Multiply by 4

$4x + 1 > -30$

Simplify

$x - 8 < -4$

Simplify

$4x + 1 - 1 > -30 - 1$

Subtract 1

$x - 8 + 8 < -4 + 8$

Add 8

$4x > -31$

Simplify

$x < 4$

Simplify

$4x/4 > -31/4$

Divide by 4

$x > -31/4$  or  $x > -7.75$





8.  $22.8 < 1.5(x-5) + .3 < 39.3$

$22.8 < 1.5x - 7.5 + .3 < 39.3$

Distribute 1.5 throughout the parenthesis

$22.8 < 1.5x - 7.2 < 39.3$

Combine like terms:  $-7.5 + .3 = -7.2$

$22.8 + 7.2 < 1.5x - 7.2 + 7.2 < 39.3 + 7.2$

Add 7.2 to all three sides

$30 < 1.5x < 46.5$

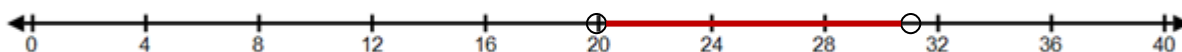
Simplify

$30/1.5 < 1.5x/1.5 < 46.5/1.5$

Divide by 1.5 on all three sides

$20 < x < 31$

Simplify



9.  $-3(x+8) - 7x > 46$  or  $4(x-2)+3x > 15$

This is a union of sets, so we must solve each inequality separately.

$-3(x+8) - 7x > 46$

OR

$4(x-2) + 3x > 15$

$-3x - 24 - 7x > 46$      Distribute

$4x - 8 + 3x > 15$      Distribute

$-10x - 24 > 46$      Combine like terms

$7x - 8 > 15$      Combine

$-10x - 24 + 24 > 46 + 24$      Add 24

$7x - 8 + 8 > 15 + 8$      Add 8

$-10x > 70$      Simplify

$7x > 23$      Simplify

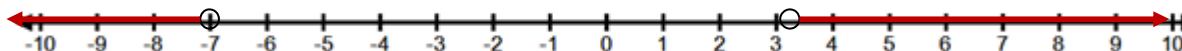
$-10x/-10 > 70/-10$      Divide by -10

$7x/7 > 23/7$      Divide by 7

$x < -7$

Reverse sign

$x > 23/7$  or  $x > 3.29$



10. Challenge:  $\frac{3}{4} < \frac{3}{4} + \frac{2}{3}(x-5) < \frac{11}{4}$

$12(\frac{3}{4}) > 12[\frac{3}{4} + \frac{2}{3}(x-5)] < 12(\frac{11}{4})$

Multiply by 12 to get rid of all fractions

$9 < 9 + 8(x-5) < 33$

Simplify

$9 < 9 + 8x - 40 < 33$

Distribute the 8 throughout the parenthesis

$9 < 8x - 31 < 33$

Combine like terms:  $9 - 40 = -31$

$9 + 31 < 8x - 31 + 31 < 33 + 31$

Add 31 to all three sides

$40 < 8x < 64$

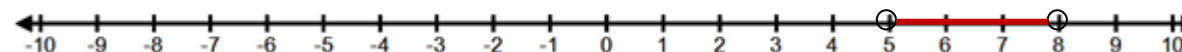
Simplify

$40/8 < 8x/8 < 64/8$

Divide by 8 on all three sides

$5 < x < 8$

Simplify





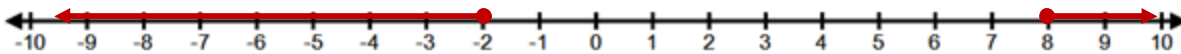
**Check Your Understanding**

For numbers 1-3, write a compound inequality for each and then graph the solution set on a number line. (2 points each)

1. all numbers greater than or equal to 8 or less than or equal to -2.

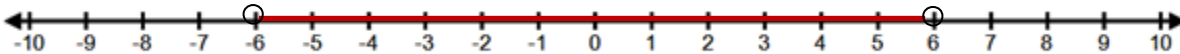
This is a union of sets since it contains the word "or".

$$x \geq 8 \text{ OR } x \leq -2$$



2. all numbers between -6 and 6.

$$-6 < x < 6$$



3. a starting salary that's at least \$35000 and not more than \$55000.

$$35000 \leq x \leq 55000$$



For numbers 4 and 5, solve each compound inequality and graph the solution on a number line.  
(3 points each)

4.  $-2/3x + 7 < 3$  or  $5(x+1) < 30$

This is a union of sets, so we must solve separately.

$-2/3x + 7 < 3$

OR

$5(x+1) < 30$

$-2/3x + 7 - 7 < 3 - 7$       Subtract 7

$5x + 5 < 30$       Distribute 5

$-2/3x < -4$       Simplify

$5x + 5 - 5 < 30 - 5$       Subtract 5

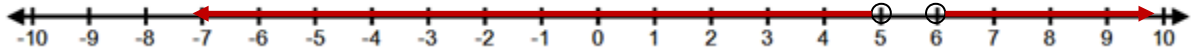
$(-3/2) \cdot (-2/3x) < (-3/2) \cdot (-4)$       Multiply by  $-3/2$

$5x < 25$       Simplify

$x > 6$

$5x/5 < 25/5$

$x < 5$



5.  $-2 > -2(x-5) > -4$

$-2 > -2x + 10 > -4$

Distribute  $-2$  throughout the parenthesis

$-2 - 10 > -2x + 10 - 10 > -4 - 10$

Subtract 10 from both sides

$-12 > -2x > -14$

Simplify

$-12/-2 > -2x/-2 > -14/-2$

Divide by  $-2$

$6 < x < 7$

Simplify

